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Transmission Line Matrix Method Reduced Order Modeling

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Abstract— **The finite-difference time-domain (FDTD) method and the transmission line matrix (TLM) method allow the formulation of state-equation representations of the discretized electromagnetic field. These representations usually involve very large numbers of state variables. Reduced order modeling (ROM) of the investigated structure may yield considerable reduction of the computational effort and can be used to generate compact models of the electromagnetic system. While complexity reduction approaches based on moment matching techniques have been intensively studied in the case of FDTD, they have not yet been considered for TLM. In this paper we apply Krylov subspace methods to TLM using the basic Arnoldi and non-symmetric Lanczos algorithm. It is shown that the inherent unitarity property of the TLM operator nevertheless implies an essential difference in comparison to former implementations for FDTD or circuit analysis. Simulation results for a rectangular cavity resonator using both TLM with and without ROM and a study of the convergence of the eigenvalues are presented here.**

Keywords— **Transmission Line Matrix (TLM) Method, Reduced Order Modeling (ROM).**

I. Introduction

The TLM has proven to be a powerful tool for solving Maxwell's equations and has been successfully applied to the analysis of various complicated planar and three-dimensional structures [1]. Like related space discretizing methods the TLM solves a system of linear equations with a very high order. Realistic problems often require a total discretization effort in the order of one million cells, leading to an even larger state space dimension and number of eigenvalues. Since higher spatial resolution in addition demands a shorter time step to guarantee stability, simulation effort may easily become expensive, if not prohibitive. A reduced order model is a system of significantly lower order that approximates the original system and sustains the relevant eigenvalues. The Krylov subspace methods for extracting the interesting parts of the eigenvalues spectrum and generating a reduced order model have been used effectively in FDTD, FEM and circuit analysis [2], [3], [4]. The TLM differs from conventional finite difference schemes, e.g. FDTD in the sense that the former is a discrete time evolution scheme. Thus reciprocity and energy conservation of the model do not yield symmetry of the discrete time evolution operator. Consequently the symmetric Lanczos algorithm cannot be applied and the use of general Arnoldi- or Lanczos-procedures becomes necessary.

II. The TLM-Method

The TLM method has emerged as a powerful method for computer modeling of electromagnetic field. The TLM-method is based on the analogy between the discretized electromagnetic field and a mesh of transmission lines. The space is discretized by subdivision into rectangular cells and the tangential components of the electromagnetic field are sampled at the center of each boundary surface of a cell. The time evolution of the electromagnetic field is modeled by wave pulses propagating between adjacent cells and scattered within the cells. The TLM cell is described by a 12-port in Fig. 1, which represents the symmetrical condensed node (SCN) and provides the respective abstract network model. The field state in TLM is represented comprisingly by an enumerable set of real quantities and the field evolution is governed by linear mapping rules. For these reasons, a Hilbert space representation of the field state and the field evolution is possible [1]. The propagation and the scattering of the wave amplitudes may be expressed by operator equations. We introduce the Hilbert space $\mathcal{H}_m$ spanned by the sequence of the grid points with spatial indices l, m and n. The ket-vectors $|l,m,n\rangle$– given in Dirac notation – represent an orthonormal base of the space $\mathcal{H}_m$. The incident wave amplitudes a_1 to a_{12} and the scattered wave amplitudes b_1 to b_{12} at a single SCN are elements of the real vector space $\mathcal{R}^{12}$. Finally we construct the state space of the wave amplitudes $\mathcal{H}_F$ from the direct product of $\mathcal{H}_m$ and $\mathcal{R}^{12}$. The vectors $|a\rangle \in \mathcal{H}_F$ and $|b\rangle \in \mathcal{H}_F$ denoting all incident and scattered wave amplitudes of the TLM mesh can thus be written in the form

$$|{}_k a\rangle = \sum_{l,m,n=-\infty}^{+\infty} {}_k\boldsymbol{a}_{l,m,n}\,|l,m,n\rangle\,,$$
$$|{}_k b\rangle = \sum_{l,m,n=-\infty}^{+\infty} {}_k\boldsymbol{b}_{l,m,n}\,|l,m,n\rangle\,, \qquad (1)$$

where the vector ${}_k\boldsymbol{a}_{l,m,n}$ and the vector ${}_k\boldsymbol{b}_{l,m,n}$ summarize the incident and the scattered wave amplitudes at the SCN (l,m,n)

$${}_k\boldsymbol{a}_{l,m,n} = {}_k\left[a_1, a_2, a_3, \ldots a_{10}, a_{11}, a_{12}\right]^T_{l,m,n}\,,$$
$${}_k\boldsymbol{b}_{l,m,n} = {}_k\left[b_1, b_2, b_3, \ldots b_{10}, b_{11}, b_{12}\right]^T_{l,m,n}\,. \qquad (2)$$

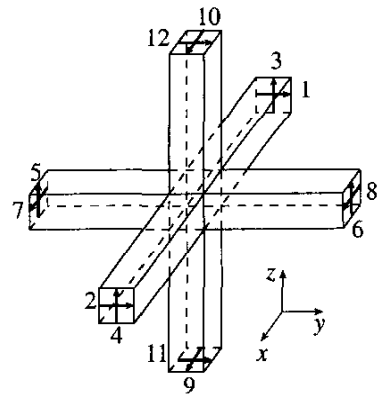


Fig. 1. A three-dimensional SCN.

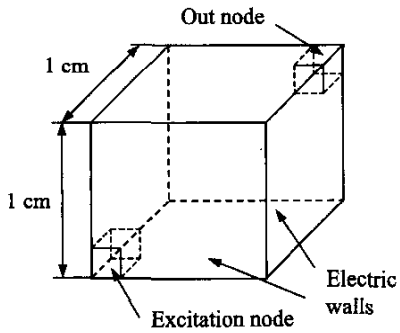


Fig. 2. Rectangular resonator.

The left index k denotes the discrete time coordinate with unit time interval Δt. Accordingly the simultaneous scattering at all TLM mesh nodes is described by the operator equation

$$|_{k+1}b\rangle = \boldsymbol{S}\,|_k a\rangle \ . \tag{3}$$

The scattering matrix $\boldsymbol{S}$ is given by

$$\boldsymbol{S} = \begin{bmatrix} 0 & \boldsymbol{S}_0 & \boldsymbol{S}_0{}^T \\ \boldsymbol{S}_0{}^T & 0 & \boldsymbol{S}_0 \\ \boldsymbol{S}_0 & \boldsymbol{S}_0{}^T & 0 \end{bmatrix} , \tag{4}$$

where

$$\boldsymbol{S}_0 = \begin{bmatrix} 0 & 0 & \frac{1}{2} & -\frac{1}{2} \\ 0 & 0 & -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 \end{bmatrix} . \tag{5}$$

In order to describe the propagation of the wave amplitudes in the TLM mesh, we define the spatial shift operators in positive $l-$ $m-$ and $n-$ direction $\boldsymbol{X}$, $\boldsymbol{Y}$, $\boldsymbol{Z}$ and their Hermitian conjugates $\boldsymbol{X}^\dagger$, $\boldsymbol{Y}^\dagger$ and $\boldsymbol{Z}^\dagger$ on the space $\mathcal{H}_m$ by

$$\begin{aligned} \boldsymbol{X}\,|l,m,n\rangle &= |l+1,m,n\rangle \ , \\ \boldsymbol{X}^\dagger\,|l,m,n\rangle &= |l-1,m,n\rangle \ , \\ \boldsymbol{Y}\,|l,m,n\rangle &= |l,m+1,n\rangle \ , \\ \boldsymbol{Y}^\dagger\,|l,m,n\rangle &= |l,m-1,n\rangle \ , \\ \boldsymbol{Z}\,|l,m,n\rangle &= |l,m,n+1\rangle \ , \\ \boldsymbol{Z}^\dagger\,|l,m,n\rangle &= |l,m,n-1\rangle \ . \end{aligned}$$

Making use of the 12×12 -matrix $(\boldsymbol{\Delta}_{i,j})_{m,n} = \delta_{i,m}\ \delta_{j,n}$ and the respective numbering of the ports (see Fig. 1), the connection operator $\boldsymbol{\Gamma}$ may be written in the matrix form

$$\begin{aligned} \boldsymbol{\Gamma} = \boldsymbol{X}(\boldsymbol{\Delta}_{1,2}+\boldsymbol{\Delta}_{3,4}) + \boldsymbol{X}^\dagger(\boldsymbol{\Delta}_{2,1}+\boldsymbol{\Delta}_{4,3}) + \\ +\boldsymbol{Y}(\boldsymbol{\Delta}_{5,6}+\boldsymbol{\Delta}_{7,8}) + \boldsymbol{Y}^\dagger(\boldsymbol{\Delta}_{6,5}+\boldsymbol{\Delta}_{8,7}) + \\ +\boldsymbol{Z}(\boldsymbol{\Delta}_{9,10}+\boldsymbol{\Delta}_{11,12}) + \boldsymbol{Z}^\dagger(\boldsymbol{\Delta}_{10,9}+\boldsymbol{\Delta}_{12,11}) \ . \end{aligned} \tag{6}$$

The complete cycle of the TLM-algorithm can be expressed as

$$|_{k+1}a\rangle = \boldsymbol{\Gamma S}\ |_k a\rangle \ . \tag{7}$$

In z-domain the state equation of the TLM-system is thus given as

$$(z\boldsymbol{I} - \boldsymbol{\Gamma S})\,|\tilde{a}\rangle = 0 \, . \tag{8}$$

It can easily be verified that the scattering matrix $\boldsymbol{S}$ as well as the connection matrix $\boldsymbol{\Gamma}$ is real, symmetric and unitary. While only the case for infinite homogenous space is shown here, these properties are also valid for arbitrary bounded and lossless structures. The unitarity of $\boldsymbol{S}$ and $\boldsymbol{\Gamma}$ passes over to the general TLM-operator $\boldsymbol{A} = \boldsymbol{\Gamma S}$ whereas symmetry is lost because the scattering and connection operator do not commute : $\boldsymbol{\Gamma S} \neq (\boldsymbol{\Gamma S})^{-1}$. The eigenvalues of $\boldsymbol{\Gamma S}$ are located on the unit circle in the complex plain and related to the corresponding eigenfrequencies f_i by

$$\lambda_i = e^{j\ 2\pi f_i \Delta t} \ . \tag{9}$$

III. Reduced order modeling

In the complexity reduction approach we search for a model of reduced order that approximates the discrete TLM model which is represented by the state equation (8) derived in the last section. Assuming a multiport excitation vector $|\tilde{a}\rangle_s$ and a vector of port responses $|\tilde{a}\rangle_r$ we obtain

$$|\tilde{a}\rangle = z^{-1}\left(\boldsymbol{A}|\tilde{a}\rangle + \boldsymbol{R}|\tilde{a}\rangle_s\right) , \tag{10}$$

$$|\tilde{a}\rangle_r = \boldsymbol{Q}|\tilde{a}\rangle \, , \tag{11}$$

and the multiport matrix impulse response can thus be calculated at as

$$\tilde{\boldsymbol{H}}_N = \boldsymbol{Q}(z\boldsymbol{I}-\boldsymbol{A})^{-1}\,\boldsymbol{R} = \sum_{k=1}^{\infty} z^{-k}\boldsymbol{Q}\,(\boldsymbol{A})^{k-1}\,\boldsymbol{R} \, . \tag{12}$$

Although the order N of the above equation is very high, a large number of eigenstates result from the discretization

are neither controllable nor observable for the given localization of excitation and response. Therefore it is meaningful to generate a reduced order model containing only those eigenstates that will significantly affect the accuracy of the solution. This may be done using moment matching techniques [5]. The basic idea is to project the system of state equations of dimension N onto a space of significantly lower dimension n such that only the first n moments in the Laurent series expansions of the original system (12) and the reduced system are matched. Using a projection operator $\boldsymbol{W}_n : \mathcal{R}^n \rightarrow \mathcal{R}^N$ and its transpose $\boldsymbol{W}_n^T : \mathcal{R}^N \rightarrow \mathcal{R}^n$ we obtain the reduced system of order $n << N$

$$\tilde{\boldsymbol{H}}_{n_1} = \boldsymbol{Q}\boldsymbol{W}(z\boldsymbol{I} - \boldsymbol{A}_n)^{-1}\boldsymbol{W}^T\boldsymbol{R} = \\ = \sum_{k=1}^{\infty} z^{-k}\boldsymbol{Q}\boldsymbol{W}\left(\boldsymbol{A}_n\right)^{k-1}\boldsymbol{W}^T\boldsymbol{R}\,. \tag{13}$$

where $\boldsymbol{W}_n$ is such that $\boldsymbol{W}_n^T\boldsymbol{W}_n = \boldsymbol{1}_n$.
From the Laurent series expansion in (12) and (13) we can observe that the reduced model will exhibit the necessary properties if we project onto the Krylov space $\mathcal{K}_n$ induced by general TLM-operator $\boldsymbol{A}$, $\boldsymbol{M}$ and n:

$$\mathcal{K}_n(\boldsymbol{A}, \boldsymbol{M}, n) = \mathrm{span}\{\boldsymbol{M}, \boldsymbol{A}\boldsymbol{M}, (\boldsymbol{A})^2\boldsymbol{M}, \ldots, (\boldsymbol{A})^{n-1}\boldsymbol{M}\}\,. \tag{14}$$

In order to orthogonalize the Krylov subspace and determine the projection matrix $\boldsymbol{W}_n$, the basic Arnoldi-algorithm has been applied. The block Arnoldi algorithm reduce the general TLM-operator to a $n \times n$ block upper Hessenberg matrix $\boldsymbol{A}_n$.
Using the block non-symmetric Lanczos algorithm the operator $\boldsymbol{A}$ can be reduced to a $n \times n$ block tridiagonal form $\boldsymbol{T}_n$ such that:

$$\boldsymbol{V}_n^T\boldsymbol{A}\boldsymbol{W}_n = \boldsymbol{D}_n\boldsymbol{T}_n\,, \tag{15}$$

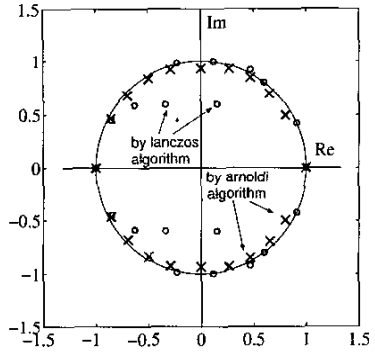
where the two matrices $\boldsymbol{V}$ and $\boldsymbol{W}$ are generated to be orthogonal, $\boldsymbol{V}_n^T\boldsymbol{W}_n = \boldsymbol{D}_n$. The corresponding multiport impulse response is:

$$\tilde{\boldsymbol{H}}_{n_2} = \boldsymbol{Q}\boldsymbol{W}_n(z\boldsymbol{I} - \boldsymbol{T}_n)^{-1}\boldsymbol{D}^{-1}\boldsymbol{V}_n^T\boldsymbol{R} = \\ = \sum_{k=1}^{\infty} z^{-k}\boldsymbol{Q}\boldsymbol{W}_n(\boldsymbol{T}_n)^{k-1}\boldsymbol{D}^{-1}\boldsymbol{V}_n^T\boldsymbol{R}\,. \tag{16}$$

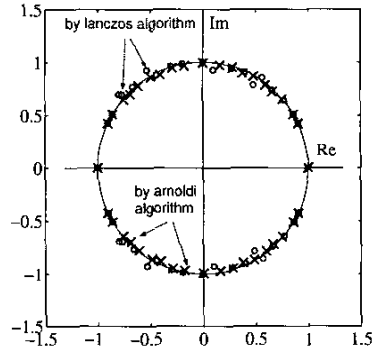
For a more detailed description of the relevant mathematics refer to [5].
An important characteristic of the depicted technique is that the operator $\boldsymbol{A}$ need not be known explicitly, as only its impact on the sequence of Krylov vectors must be taken into account. Subsequently the computation of the reduced model can be integrated in the usual iterative TLM-algorithm in a very efficient manner. Indeed the choice of a useful iterative eigenvalue solver is determined by the properties of the matrix $\boldsymbol{A}$. Due to its missing symmetric property, the general non-symmetric Lanczos or Arnoldi algorithm will have to be employed. Compared with a symmetric Lanczos algorithm this is disadvantageous in terms of both numerical efficiency and stability. On the other hand the unitarity of $\boldsymbol{A}$ represents an a priori knowledge of the possible eigenvalue distribution and can thus be used to identify spurious eigenvalues.

IV. Simulation results

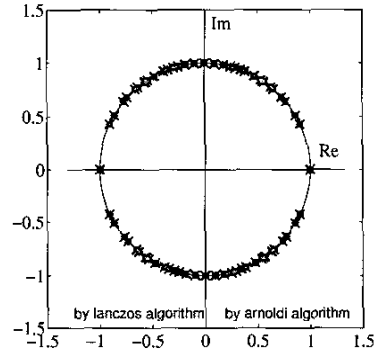
We consider a rectangular air-filled cubic 1cm resonator with ideally conducting walls. Excitation node and observation point are placed at opposite edges of the resonator according to Fig. 2.



(a) at 20 algorithm steps



(b) at 40 algorithm steps



(c) at 70 algorithm steps

Fig. 3. Eigenvalues of $\boldsymbol{A}$ operator calculated for 5 cell diskretization at different algorithm steps.

The structure is investigated for different discretizations of 3, 5 or 10 cells in each direction – corresponding to state

space dimensions of 324, 1500 and 12000, respectively. The eigenfrequencies of the original system are determined by Fourier transformation of the TLM time domain response of a Dirac excitation. For the 5-cell discretization case, at 70 Arnoldi or Lanszos algorithm steps the all calculated eingenvalues λ_i of $\boldsymbol{A}$ operators are placed on the unit circle (Fig. 3a). For a smaller number of steps the Arnoldi algorithm approximate the eigenvalue spectrum of the general TLM operators more exactly (Fig. 3a,b), due ti its quicker convergence.

Converged eigenvalues can almost certainly be recognized by their position close to the unit circle in the complex plane. For these eigenvalues the corresponding eigenfrequencies f_i are calculated according to equation (9).

From Fig. 4a a close relationship between the simulated eigenfrequencies and the theoretical resonances can be observed even for a rough discretization of $3 \times 3 \times 3$ cells. For a good agreement between the reduced model and the original TLM it is sufficient to perform 20 steps using both algorithms.

Fig. 4a and Fig. 4b show that in the case of discretizations of 3 and 5 cells all eigenfrequencies related to physical modes of the resonator are identified at 20 and 70 Arnoldi steps, respectively.

We determine the number of Arnoldi/Lanczos steps required to find all the relevant eigenfrequencies. It turns out that the ratio of necessary steps and state space dimension decreases significantly with increasing complexity of the structure. In all cases the reduction of the operator operator $\boldsymbol{A}$ by a factor of at least 250 is achieved.

The implementation of the Lanczos algorithm is more efficient, since the computational time increases with the number of steps n in contrast to n^2 by Arnoldi.

V. Conclusion

We considered the application of complexity reduction to the TLM method. It was shown that the non-symmetric property of the matrix $\boldsymbol{\Gamma S}$ requires the use of a general non-symmetric eigenvalue solver. The basic Arnoldi and the non-symmetric Lanczos methods based approach were successfully applied to a TLM model of a rectangular cubic resonator. We observed a rapid convergence of the relevant eigenvalues and a convergence of both Arnoldi and Lanczos algorithms for simulation mesh sizes up to 12000 cells. A significant reduction on the general TLM operators allow to generate a compact model of a electromagnetic system. On the other hand, the numerical effort for the Arnoldi algorithm seems to present a serious drawback for the application to problems of higher dimensions. As a consequence, further investigation and implementation of the more efficient non-symmetric Lanczos algorithm for structures with losses will will be performed.

VI. Aknowledgement

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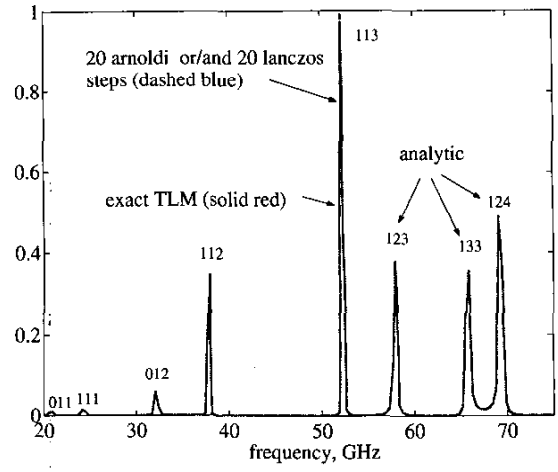
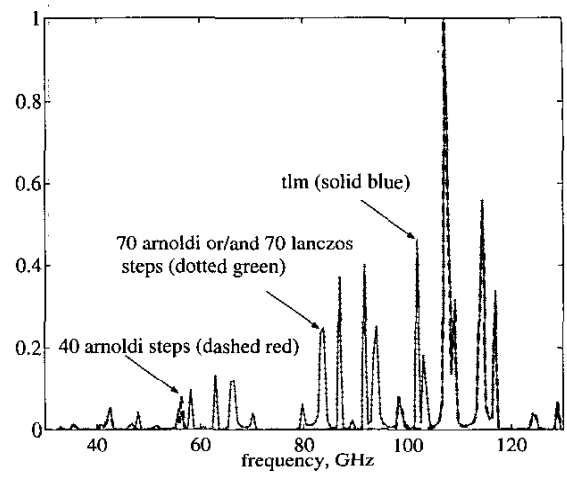


(a) for 3 cell discretization



(b) for 5 cell discretization

Fig. 4. Spectrum of the cavity resonator calculated by TLM with and without ROM. The analytic eigenfrequencies are denoted by the mode.

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